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"On Some Iterative Methods for Fractional Differential Equations"

Research proposal submitted

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“On Some Iterative Methods for Fractional Differential Equations”

I. Introduction to proposed research topic:

In recent years, fractional order calculus has been studied as alternative calculus in mathematics and it was applied in various field in science and engineering. Numerous problems in Physics, Chemistry, Biology and Engineering can be models with fractional derivatives. Now the most of modeled fractional differential equations donot have analytic solutions. Therefore, numerical techniques are used intensively, recently the adomian decomposition method, homotopy perturbation method and variational iterative methods have been used for solving wide range of problems. In our work we shall develop new iterative methods such as fractional order Adomian decomposition method, fractional order variational iteration method and explicit fractional order finite difference methods to obtain the numerical solutions of fractional differential equations.

For example, we will solve the following types of fractional partial differential equations

$$\frac{\partial^{\alpha} \mathrm{u}}{\partial \mathrm{t}^{\alpha}} = \alpha \frac{\partial^{2} \mathrm{u}}{\partial x^{2}} + \mathrm{f}\left(x,\ \mathrm{t}\right) \qquad 0 < x < \mathrm{L},\ 0 < \alpha \leq 1,\ \mathrm{t} > 0$$

Subject to IC : $\mathrm{u}(x, 0) = \mathrm{f}(x),\ 0 \leq x \leq \mathrm{L}$

and BCS : $\mathrm{u}(0, \mathrm{t}) = g_1(\mathrm{t}),\ \mathrm{u}(\mathrm{L}, \mathrm{t}) = g_2(\mathrm{t}),\ \mathrm{t} > 0$

II. Purpose of carrying out proposed research work:

Differential equations play a major role in Mathematics, Physics and Engineering. The purpose of the research will develop new iterative methods of fractional differential equations to obtain their solutions. Also, we will represent graphically. The solution of fractional differential equations obtains by new iterative methods using mathematical software Mathematica.

Also we will study Numerical methods to obtain the solutions of fractional partial differential equations, such as Explicit methods. We will compare the solutions obtain by Decomposition methods and Numerical methods.

III. Survey or Review of literature and development in the relevant subject

A numerical method based on the Adomian Decomposition Method (ADM) which has been used from the 1970's to the 1990's by George Adomain, The Differential Transform Method (DTM) has been successfully developed by Zhou (1986) in electric circuit analysis. DTM has been used to solve linear and non-linear differential equations.

ADM and DTM has been shown to solve effectively, easily and accurately a large class of linear and non-linear, ordinary, partial deterministic or stochastic differential equations with approximate solutions which converge rapidly to accurate solutions. The basic motivation of the work is to apply the ADM and DTM to the ordinary differential equations (ODE) and partial differential equations (PDE). It is well known now in the literature that this algorithm provides the solution in a rapidly convergent series. ADM and DTM are very effective and convenient for solving multi-pantograph equations.

Variational Iteration Method (VIM) is first proposed by Ji-Huan He. In the year 1998, to solve fractional differential equations with great success following the above idea, Draganescu, Momani and Odibate applied VIM to more complex fractional differential equations, showing the effectiveness and accuracy of the method.

In 2002, the Adomain Decomposition Method (ADM) was suggested to solve fractional differential equations but many researches found it very difficult to calculate the Adomain polynomials. In 2007,

Momani and Odibate applied the Homotopi - Perturbation Method (HPM) to fractional differential equations and show that HPM is an alternative methods for obtaining the solutions of fractional differential equations.

The another powerful method is called the Variational Iteration Method (VIM) has been successfully applied to many situations.

Recently, a new iterative method was presented by Daftardar – Gejji and Jafari. This technique solves many types of non linear equations such as ordinary and partial differential equations of integer and fractional order. Jafari et al applied. This method to obtain the solution of linear / non-linear diffusion and wave fractional equations. Daftardar – Gejji and Bhalekar used it for solving fractional boundary value problems and evolution equations.

Numerical methods and analysis of the fractional order partial differential equations are limited to date. Some different numerical methods for solving the space or time fractional partial differential equations have been proposed Liu et al transformed the space fractional partial differential equation in to a system of ordinary differential equations (Method of Lines) which was then solved using backward differentiation formulae.

Shen et al proposed an explicit finite difference approximation for the space fractional diffusion equation The stability and convergence of the method is discussed Lin and Liu proposed the high order approximation of the fractional ordinary differential equation and discussed the convergence and stability of the fractional high order methods.

IV. Research gap:

Most important thing is that many fractional differential equations do not have an analytical solution. Therefore, we develop the numerical

methods to obtain there solutions. Also, to discuss the stability and convergence of the numerical methods.

Also wide scope in to obtain exact solution of fractional differential equations by developed new iterative techniques and it will represent graphically.

V. Importance of the Study:

Fractional calculus is becoming a useful and key tool in the analysis of scientific problems in Physics, Engineering, Biology and Economics, Fractional formalism to be useful in solving practical problems, fractional calculus, integration and differentiation of an arbitrary or fractional order, provides new tools that expand the descriptive power of calculus beyond the familiar integer-order concept of rate of change and area under the curve.

a) **International Status:** Many scientists and researchers many more contributed in the development of fractional differential equations and Fractional calculus like, Leibniz, Euler, Lagrance, Laplace, Fourier, Abel, Liouvielli, Grunwald, Riemann, Riesz, Miller, Ross, Igor Prodlubny, Richtmeyer, Morton, Smith, Anderson, Roache etc.

b) **National Status:** Indian's also contributed remarkable in the development of fractional calculus like R.K. Sexana, V. Srivastava, K.N. Rai, S. Das, K. Gupta, S. Saha Roy, V.B.L. Chawrasia, D. Kumar, Pradeep Kumar, D. Bahuguna etc.

c) **Regional Status:** Many researchers from Maharashtra have been actively involve in the study of fractional calculus. Like Kiran, M. Kolwankar, A.K. Pani, PSV Nataraj, S. Bhalekar, V. Daftardar-Gejji, K.C. Takale, J. Verma, D.B. Dhaigude etc.

d) **Scope of Study:** In recent years, their has been a greatdeal of interest in fractional differential equations.

During the last decade, fractional calculus has been applied to almost every field of Science, Engineering and Mathematics. Many application of fractional calculus can be found in turbulence and fluid dynamics, stochastic dynamical system, plasma physics and controlled thermonuclear fusion, nonlinear control theory, image processing, nonlinear biological systems and astrophysics. There are few iterative methods for obtaining the solution of fractional differential equation (FDE), therefore, we develop new iterative methods to obtain their solutions and discuss the stability and convergence of the numerical methods upto the length.

VI. Objectives of Proposed Research Work:

1) To study ordinary and partial differential equations.
2) To study different iterative methods for differential equations which are available in the literature.
3) To study basic concept of fractional calculus.
4) To study fractional differential equation.
5) To develop iterative methods for fractional differential equations.
6) To obtain the approximate solutions of fractional differential equations by developed new iterative techniques and it will represented graphically by Mathematica software.
7) To study recent applications of fractional differential equations.
8) To publish of our work in reputed journals.

VII. Hypothesis:

The proposed work is to prove that how mathematics is useful in other branches of science. We hope that our research work will be useful in applied sciences and engineering field.

Hence, in the present work, we will try to develop the new iterative techniques for fractional differential equation and to obtain their solution. These will represented graphically by Mathematica software.

VIII. Limitation of the Work:

1) The fractional order model are more adequate than integer order model.
2) Fractional derivatives and integrals enable the description of memory properties of various materials and processes.
3) Most of the fractional differential equations do not have analytical solutions.

IX. Research Methodology:

The following steps will be including in the methodology.

1) To refer the relevant reference books on partial differential equation and fractional partial differential equation.
2) To study Mathematica software.
3) To study fractional calculus.
4) To study Iterative / Numerical methods for obtaining approximate solutions of fractional differential equations.
5) To discuss the stability and convergence of Iterative / Numerical methods.
6) To represent Numerical solutions of fractional differential equations graphically by Mathematica and discuss their nature.

X. Expected outcome:

1) We will develop new iterative methods for fractional differential equations.
2) We will develop numerical methods for fractional partial differential equations.

3) To find the approximate solutions of fractional differential equations by developed new iterative technique and it will represented graphically by Mathematica software.

XI. Year wise plan of work and targets to be achieved:

First year plan :

1) To refer the reference books on differential equation and fractional differential equation.
2) To develop different Iterative methods, Numerical methods for differential equation.
3) To obtain the solutions of partial differential equations by iterative methods.
4) To visualize solutions by graphs using a powerful software Mathematica.

Second year plan :

1) To study fractional calculus.
2) To study fractional differential equations.
3) To develop iterative methods for obtain the solution of fractional differential equations.
4) To visualize the solutions by graphs using Mathematica and discuss the nature.
5) To discuss the stability and convergence of iterative methods.

Third year plan :

1) To develop Numerical methods for fractional partial differential equations.
2) To discuss stability and convergence of the numerical methods.
3) We will publish our work in reputed journals.
4) Submission of completed work to concerned authority.

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